2.2: Describing Sets

Definition: A set is any collection of objects with no repetitions. An object in a set is said to be an element of the set. One way to write a set is to list them in { } with commas in between the elements.

Notation: If A is a set and a is an element of A, we write $a \in A$. If b is not an element of A, we write $b \notin A$.

Example: Write the set of the first five counting numbers and give examples of elements in and not in the set.

Definition: (Set builder notation) Let S be a set. Then we can write $S = \{x \mid x \text{ satisfies some conditions}\}$. This is read "$S$ equals the set of elements x such that x satisfies some conditions".

Another way to think of set builder notation is {form of elements | conditions}. This will show up more in the examples.

Example: Write $S = \{1, 2, 3, 4, 5\}$ in set builder notation.

Definition (Special Sets):

(1) The Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, ...\}$
(2) The Integers: $\mathbb{Z} = \{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$
(3) The Real Numbers: $\mathbb{R} = \{x \mid x$ is any number that can be written as a decimal$\}$

Example: Use set builder notation to write the set of all real numbers between 0 and 1.

Example: Use set builder notation to write the set of all even integers.

Example: Use set builder notation to write the set of perfect squares 1, 4, 9, 16, etc.

Example: Describe the elements of the following sets.

(a) $\{3x \mid x \in \mathbb{Z}\}$

(b) $\{-x \mid x \in \mathbb{N}\}$

(c) $\{a/b \mid a, b \in \mathbb{Z}, b \neq 0\}$

Definition: Two sets are <u>equal</u> if they contain exactly the same elements in any order.

Definition: The <u>cardinal number</u> of a set S, denoted $n(S)$ or $|S|$, is the number of elements of S.

Definition: The <u>empty set</u>, denoted $\varnothing$, is the set with no elements. The empty set can also be written as { }.

Definition: A set is a <u>finite set</u> if the cardinal number of the set is 0 or a natural number. A set with infinitely many elements, such as the natural numbers, is called an <u>infinite set</u>.

Example: Find the cardinal number of $A = \{1, 2, 3, 4\}$, $B = \{0\}$, $C = \{2, 4, 6, 8, ...\}$, and $\varnothing$.

Example: Find the cardinal number of the following sets.

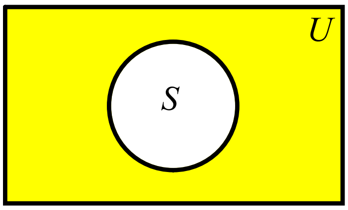
(a) $S = \{1, 4, 7, 10, 13, ..., 40\}$

(b) $T = \{33, 37, 41, 45, 49, ..., 353\}$

Definition: The <u>universal set</u>, denoted U, is the set of all elements being considered in a given discussion.

Definition: The <u>complement</u> of a set S, denoted $\overline{S}$, is the set of all elements in U that are not in S. That is, $\overline{S} = \{x \mid x \in U \text{ and } x \notin S\}$.

A complement can be thought of in the following manner. The shaded region is $\overline{S}$:

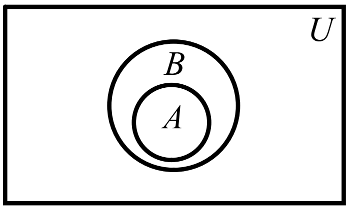


Example: If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, find the complements of $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 4, 6, 7\}$.

2.2 Notes

Definition: If A and B are sets, we say that A is a subset of B, denoted $A \subseteq B$, if every element of A is an element of B. If $A \subseteq B$ and $A \neq B$, we say that A is a proper subset of B, denoted $A \subset B$.

A subset can be thought of in the following manner. In the figure, $A \subseteq B$:



Example: Fill in the blanks with either $\subseteq$ or $\not\subseteq$.

$\{1, 2, 3, 4\}$ ___ $\{1, 2, 3, 4, 5\}$

$\{1, 2, 3, 4\}$ ___ $\{1, 2, 3, 4\}$

$\{1, 2, 3, 4, 5\}$ ___ $\{1, 2, 3, 4\}$

$\{1, 2, 3, 4, 5\}$ ___ $\{1, 2, 3, 4, 6\}$

$\{0\}$ ___ $\{1, 2, 3, 4\}$

$\varnothing$ ___ $\{1, 2, 3, 4\}$

$\{1, 2, 3, 4\}$ ___ $\varnothing$

$\varnothing$ ___ $\varnothing$

Example: Fill in the blanks with either $\in$, $\notin$, $\subseteq$, or $\not\subseteq$.

$\{2\}$ ___ $\{1, 2, 3\}$

0 ___ $\mathbb{N}$

2 ___ $\{1, 2, 3\}$

$\mathbb{Z}$ ___ $\mathbb{N}$

5 ___ $\{1, 2, 3, 4\}$

5 ___ $\{2x \mid x \in \mathbb{Z}\}$

$\varnothing$ ___ $\{1\}$

$\mathbb{R}$ ___ $\mathbb{R}$

0 ___ $\varnothing$

$\{a / b \mid a, b \in \mathbb{Z}, b \neq 0\}$ ___ $\mathbb{R}$

$\{4\}$ ___ $\{2\}$

$\{1.5\}$ ___ $\mathbb{N}$